Review Guide III
Math 161 - Fall 2014

1. Determine if the following statements are true or False. If true justify your answer. If false, provide a counterexample.
   (a) If $f'(c) = 0$, then f has a local maximum or minimum at c.
   (b) If f has a local maximum at c, then f has an absolute maximum at c.
   (c) If $f(x)$ is an odd function, then $f'(x)$ is an odd function.
   (d) If $f(x)$ is an even function, then $f'(x)$ is an odd function.

2. Find the critical numbers of the function.
   (a) $f(x) = x^3 - 12x^2 + 48x$
   (b) $g(x) = \cos x + \sin x$
   (c) $h(x) = \dfrac{x-1}{x^2 - x + 1}$

3. Find the critical number of the following functions on the given interval.
   (a) $f(x) = 2\cos x + \sin(2x)$, on $[0, 2\pi]$.
   (b) $g(t) = t + \cot\left(\dfrac{1}{2}t\right)$ on $[\frac{\pi}{4}, \frac{7\pi}{4}]$

4. Find the absolute maximum and absolute minimum of the valus of f on the given interval.
   (a) $f(x) = x^3 - 6x^2 + 9$, $[-3, 3]$
   (b) $f(x) = \dfrac{3x-4}{x^2+1}$, $[2, 4]$

5. State
   - Extreme Value Theorem.
   - Rolle's Theorem.
   - Mean Value Theorem.

6. Show that the equation $2x + \cos x = 0$ has exactly one root.

7. Show that the equation $3x + 2\cos x + 5 = 0$ has exactly one root.

8. Use the Mean value Theorem to prove the inequality

$$|\sin a - \sin b| \leq |a - b|$$

for all a and b.

9. If $f(3) = 7$, $f'(x) \geq 3$ for all x, how large can $f(8)$ possibly be?

10. Does there exist a function such that $f(0) = -1$, $f(2) = 4$ and $f'(x) \leq 2$ for all x?

11. Find the intervals on which the function is increasing or decreasing, the local maximum and minimum values of the function, intervals of concavity and the inflection points.

    (a) $f(x) = x^5 - 80x$

    (b) $g(x) = x^{\frac{1}{3}}(x + 4)$

    (c) $h(x) = \sqrt{x^2 + 1} - x$

    (d) $j(x) = \dfrac{x^2 - 4}{x^2 + 4}$

    (e) $p(x) = x + \cos x$

12. Use the guidelines (discussed in class) to sketch the following curves

    (a) $f(x) = x^3 - 12x + 2$

    (b) $g(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

    (c) $h(x) = \dfrac{1}{x^2 - 9}$

13. Find the dimensions of a rectangle with area 400 m^2 whose perimeter is as small as possible.

14. If $C(x) = 4000 + 125x - 0.4x^2 + 0.0001x^3$ is the cost function and $p(x) = 425 - 1.75x$ is the demand function, find the production level that will maximize profit.

15. Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest from the point $(1, 0)$.

16. A right circular cylinder is inscribed in a cone with height 30 cm and radius 18 cm. Find the largest possible volume of such a cylinder.

17. Find the most general antiderivative of the function.

    (a) $f(x) = 8x^7 + 6x^5 + 3x^2$

    (b) $g(x) = \sqrt[4]{x^3} + x^2\sqrt{x}$

    (c) $h(x) = \sin x + \cos x + \sec^2 x$

18. Find f.

    (a) $f'(x) = x^3 - 4x^2 + 6x,\ f(1) = 13.$

    (b) $f''(x) = 8x^3 + 5,\ f(1) = 0,\ f'(1) = 6.$

    (c) $f''(x) = \sin x + \cos x,\ f'(0) = 3, f(0) = 4.$

19. A particle moves in a straight line and has acceleration given by $a(t) = t^2 + 6$. Its initial velocity is $-10m/s$ and its initial displacement is $-18m$. Find its position function $s(t)$.

20. A ball is thrown upward with a speed of $10m/s$ from the edge of a cliff $200m$ above the ground. When does the ball reach its maximum height? When does it hit the ground?